

Geodesics and Newton's Law in Brane Backgrounds

W. Mück*, K. S. Viswanathan† and I. V. Volovich‡

Department of Physics, Simon Fraser University, Burnaby, B.C., V5A 1S6 Canada

February 1, 2008

Abstract

In brane world models our universe is considered as a brane imbedded into a higher dimensional space. We discuss the behaviour of geodesics in the Randall-Sundrum background and point out that free massive particles cannot move along the brane only. The brane is repulsive, and matter will be expelled from the brane into the extra dimension. This is rather undesirable, and hence we study an alternative model with a non-compact extra dimension, but with an attractive brane embedded into the higher dimensional space. We study the linearized gravity equations and show that Newton's gravitational law is valid on the brane also in the alternative background.

*E-mail address: wmueck@sfu.ca

†E-mail address: kvisawana@sfu.ca

‡Permanent address: *Steklov Mathematical Institute, Gubkin St. 8, 117966 Moscow, Russia,*
volovich@mi.ras.ru

1 Introduction

It has recently been suggested by Randall and Sundrum [1] that four-dimensional gravity can arise at long distances on a brane embedded in a five-dimensional anti-de Sitter space. In their model the fifth dimension is non-compact. An effective dimensional reduction occurs because the metric perturbations admit a bound state solution which looks like a four-dimensional graviton bound to the brane. Earlier work appeared in [2, 3, 4]. This interesting alternative to compactification has been discussed in a number of recent papers [5]–[29].

The metric of the Randall-Sundrum (RS) background has the form

$$ds^2 = e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (1)$$

where $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ ($\mu, \nu = 0, 1, 2, 3$). It was argued in [1] that the Kaluza-Klein excitations, although they are light, are suppressed near the brane and almost decouple from the matter fields. Moreover, it is assumed that matter fields are trapped to the brane by a certain mechanism.

If the brane describes our Minkowski space-time, then there should exist trajectories for free massive particles located on the brane only. However, this is not true for the metric (1). The trajectory of the free particle in the metric (1) has the form (see Sec. 2 for details, here we take $y_0 = \dot{y}_0 = 0$)

$$x^\mu = x_0^\mu + v^\mu t, \quad y = \frac{1}{2k} \ln(1 - v^2 k^2 t^2) \quad (2)$$

Therefore, free massive ($v^2 < 0$) particles cannot move in Minkowski space-time without being inevitably expelled into the y -dimension. This seems rather undesirable, even if for a very small (Planck scale) k the time needed for a significant deviation in the fifth direction will be rather large. Thus, in the RS background, some other, non-gravitational mechanism is needed in order to trap matter on the brane. The simplest way to obtain an attractive brane would be to change the sign of the brane tension, although one might argue that this would imply other undesirable features. This alternative was considered in [3, 30, 31, 32].

In the present paper, we shall study the RS background and the alternative possibility, whose background metrics are given by

$$ds^2 = e^{\mp 2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2. \quad (3)$$

The upper or lower signs correspond to the RS and alternative backgrounds, respectively. This convention shall be used throughout this paper, and k is positive, $k > 0$. The metric (3) is a solution to Einstein's equation for the action

$$S = \int d^4x \int dy \sqrt{-g} (R - 2\Lambda) + \sigma \int_{y=0} d^4x \sqrt{-g_B}, \quad (4)$$

where the cosmological constant and brane tension are

$$\Lambda = -6k^2, \quad \sigma = \mp 12k. \quad (5)$$

We shall now give a brief outline of the rest of the paper. First, in Sec. 2 we study the geodesics in the two backgrounds and find that only in the alternative background gravity

provides a mechanism for the trapping of matter on the brane. Secondly, in Sec. 3 we consider the linearized gravity equations, which shall be used in Sec. 4 to derive the Newtonian limit for gravity on the brane. We find that in both backgrounds the gravitational potential for a static point source will be $\sim -1/r$, and we find an exact formula for the corrections. In Sec. 5, we give expressions for the graviton modes in the both backgrounds. Space-like modes are absent in the RS background, but our results are inconclusive for the alternative background.

2 Geodesics

In this section, we explicitly solve the geodesic equation in the RS and alternative backgrounds. We shall find that, in the RS background, ordinary matter will be expelled from the brane, but in the alternative background, the brane is attractive.

Using the zeroth order terms of the connections given in the appendix, the geodesic equation takes the form

$$\frac{d^2 x^\mu}{d\theta^2} \mp 2k \operatorname{sgn} y \frac{dx^\mu}{d\theta} \frac{dy}{d\theta} = 0, \quad (6)$$

$$\frac{d^2 y}{d\theta^2} \pm k \operatorname{sgn} y e^{\mp 2k|y|} \eta_{\mu\nu} \frac{dx^\mu}{d\theta} \frac{dx^\nu}{d\theta} = 0. \quad (7)$$

We start by integrating eqn. (6), which yields

$$\frac{dx^\mu}{d\theta} = v^\mu e^{\pm 2k|y|}, \quad (8)$$

where v^μ is a constant four-vector. Eqn. (7) is explicitly solved by the first integral of the geodesic equation,

$$\left(\frac{dy}{d\theta}\right)^2 + e^{\mp 2k|y|} \eta_{\mu\nu} \frac{dx^\mu}{d\theta} \frac{dx^\nu}{d\theta} = C,$$

where C is a constant. Hence, inserting eqn. (8), we find

$$\left(\frac{dy}{d\theta}\right)^2 + e^{\pm 2k|y|} v^2 = C, \quad (9)$$

where $v^2 = v^\mu v^\nu \eta_{\mu\nu}$.

It is convenient to change the parameterization to a non-affine parameter t such that

$$\frac{dt}{d\theta} = e^{\pm 2k|y|}.$$

Then, eqn. (8) becomes (notation $\dot{x} = dx/dt$)

$$\dot{x}^\mu = v^\mu \quad \Rightarrow \quad x^\mu = x_0^\mu + v^\mu t, \quad (10)$$

which shows that we can choose a reference frame such that $t = x^0$, i.e. t is the time on the brane. Moreover, eqn. (9) becomes

$$\dot{y}^2 e^{\pm 4k|y|} + e^{\pm 2k|y|} v^2 = C, \quad (11)$$

and we can determine C from the initial data:

$$C = \dot{y}_0^2 e^{\pm 4k|y_0|} + v^2 e^{\pm 2k|y_0|}. \quad (12)$$

Before integrating eqn. (11), we note that it depends only on $|y|$, as long as we do not pass through $y = 0$. Therefore, it is sufficient to consider $y > 0$; replacing y with $|y|$ at the end will take care of the case $y < 0$. For $y > 0$, we can integrate eqn. (11) and find

$$\sqrt{C - v^2 e^{\pm 2ky}} = \pm v^2 k t + \sqrt{C - v^2 e^{\pm 2ky_0}},$$

where we have again expressed the integration constant by the initial data. Notice, that the \pm sign in front of the term containing t on the right hand side is not (yet) related to the sign in the exponentials, but stems from the ambiguity of taking a square root. After some simple steps involving the substitution of C from eqn. (12) we obtain

$$e^{\pm 2ky_0} - e^{\pm 2ky} = v^2 k^2 t^2 \pm |\dot{y}_0| 2k e^{\pm 2ky_0} t.$$

From the initial data we can deduce that we have to replace $\pm |\dot{y}_0|$ by $\mp \dot{y}_0$. Thus, the final result is

$$e^{\pm 2k|y|} = e^{\pm 2k|y_0|} \pm 2k \dot{y}_0 e^{\pm 2k|y_0|} t - v^2 k^2 t^2. \quad (13)$$

The solution (13) is valid, as long as $|y| \neq 0$.

If we hit the brane at $y = 0$, we have to match a solution for $y > 0$ with a solution for $y < 0$. However, from eqn. (7) we see that the velocity \dot{y} must be continuous at $y = 0$, since the second term in that equation is finite. Thus, the brane will not deflect particles gravitationally, but we might expect that non-gravitational interactions with matter on the brane might do so.

The interpretation of the solution (13) is rather simple: In the RS background, which corresponds to the upper sign, ordinary matter ($v^2 < 0$) is expelled from the brane. On the other hand, tachyonic particles ($v^2 > 0$) are attracted to the brane, whereas massless particles are not affected by its presence. Using the lower sign, the brane attracts ordinary matter. This is sketched in Fig. 1.

Another interesting fact is that, for the alternative background, $e^{-2k|y|} = 0$ corresponds to $|y| = \infty$. Thus, tachyonic particles will be expelled to $|y| = \infty$ in finite brane time, as will massless particles with the right initial conditions. Moreover, there exist initial conditions for ordinary particles, which will yield $|y| = \infty$ in finite brane time.

3 Linearized Gravity

In this section, we shall study the linearized gravity equations with two applications in mind: The derivation of Newton's law on the brane and the study of graviton modes, which will be carried out in Secs. 4 and 5, respectively.

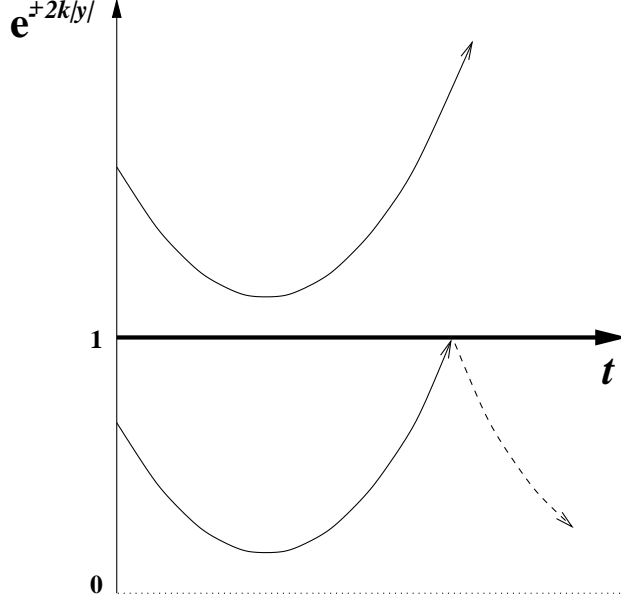


Figure 1: The solution (13) for ordinary matter, $v^2 < 0$. The region $e^{\pm 2k|y|} > 1$ corresponds to the RS background, the region $e^{\pm 2k|y|} < 1$ to the alternative background. In both cases, the brane sits at $e^{\pm 2k|y|} = 1$.

For our purpose, we introduce a matter perturbation on the brane,

$$\delta T_{00} = \delta(y)t_{00}(x), \quad (14)$$

and solve the linearized gravity equations for this source.

The form (3) of the background metric suggests to use the time slicing formalism [33] for calculating metric perturbations, although we do not slice with respect to time, but with respect to the transverse coordinate y . Let us first give some useful formulae. In the time slicing formalism, we split up the metric tensor as

$$g_{ab} = \begin{pmatrix} g_{\mu\nu} & n_\nu \\ n_\mu & n_\mu n^\mu + n^2 \end{pmatrix}, \quad g^{ab} = \frac{1}{n^2} \begin{pmatrix} n^2 g^{\mu\nu} + n^\mu n^\nu & -n^\nu \\ -n^\mu & 1 \end{pmatrix}, \quad (15)$$

where $a, b = 0, 1, 2, 3, 5$, $x^5 = y$, and $g_{\mu\nu}(x, y)$ are the induced metrics in the hypersurfaces with internal coordinates x^μ . The quantities n and n^μ are called lapse function and shift vector, respectively, and are fixed to their respective background values in the radiation gauge:

$$n^\mu = 0, \quad n = 1. \quad (16)$$

Thus, we consider a metric of the form

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + dy^2. \quad (17)$$

Then, the second fundamental form measuring the extrinsic curvature on the hypersurfaces is given by

$$H_{\mu\nu} = \frac{1}{2n} \left(\frac{\partial}{\partial y} g_{\mu\nu} - \nabla_\mu n_\nu - \nabla_\nu n_\mu \right) = \frac{1}{2} \frac{\partial}{\partial y} g_{\mu\nu}, \quad (18)$$

where ∇_μ is the covariant derivative on the hypersurfaces, and the second equality holds in the radiation gauge.

Einstein's equation is

$$R_{ab} - \frac{1}{2}g_{ab}R = -g_{ab}\Lambda + 8\pi T_{ab}, \quad (19)$$

where $T_{ab} = \bar{T}_{ab} + \delta T_{ab}$, and \bar{T}_{ab} is the background from the brane, whose non-zero components are found from eqn. (4) as

$$\bar{T}_{\mu\nu} = \mp \frac{3k}{4\pi} \delta(y) g_{\mu\nu}. \quad (20)$$

One can observe from eqns. (14) and (20) that $T_{a5} = 0$. Therefore, by virtue of the Gauss-Codazzi equations [33], the normal and mixed components of eqn. (19) become

$$\hat{R} + H_\nu^\mu H_\mu^\nu - H^2 = 2\Lambda, \quad (21)$$

$$\nabla_\mu H - \nabla_\nu H_\mu^\nu = 0, \quad (22)$$

respectively, where $H = H_\mu^\mu$, and \hat{R} is the intrinsic scalar curvature of the hypersurfaces. The tangential components of eqn. (19) simply read

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -g_{\mu\nu}\Lambda + 8\pi T_{\mu\nu}. \quad (23)$$

Eqn. (23) is the equation of motion for $g_{\mu\nu}$, whereas eqns. (21) and (22) are constraints.

We linearize eqns. (21)–(23) around the background (3), for which purpose we use an induced metric of the form

$$g_{\mu\nu} = e^{\mp 2k|y|} (\eta_{\mu\nu} + \gamma_{\mu\nu}), \quad (24)$$

where γ is a small perturbation compared to η . The indices of γ shall be raised and lowered using the Lorentz metric η . Some useful expressions for the connections and curvatures are given in the appendix.

Eqns. (21)–(23) take the forms

$$e^{\pm 2k|y|} (\gamma^{\mu\nu}_{,\mu\nu} - \square \gamma) \pm 3k \operatorname{sgn} y \gamma_{,y} = 0, \quad (25)$$

$$\frac{1}{2} \partial_y (\gamma_{,\mu} - \gamma^\nu_{\mu,\nu}) = 0, \quad (26)$$

$$\begin{aligned} & \frac{1}{2} (\gamma^\mu_{\rho,\mu\nu} + \gamma^\mu_{\nu,\mu\rho} - \square \gamma_{\nu\rho} - \gamma_{,\nu\rho}) - \frac{1}{2} \eta_{\nu\rho} (\gamma^{\mu\lambda}_{,\mu\lambda} - \square \gamma) \\ & + e^{\mp 2k|y|} \left[-\frac{1}{2} \gamma_{\nu\rho,y} + \frac{1}{2} \eta_{\nu\rho} \gamma_{,yy} \pm 2k \operatorname{sgn} y (\gamma_{\nu\rho,y} - \eta_{\nu\rho} \gamma_{,y}) \right] = 8\pi \delta T_{\nu\rho}, \end{aligned} \quad (27)$$

where the background has been cancelled using eqns. (5) and (20).

Let us start by solving the constraints. First, from eqn. (26) we find

$$\gamma^\nu_{\mu,\nu} = \gamma_{,\mu} + \xi_\mu(x), \quad (28)$$

where ξ_μ are functions of the brane coordinates x^μ only. Secondly, after substituting eqn. (28), eqn. (25) leads to

$$e^{\pm 2k|y|} \xi_{,\mu}^\mu \pm 3k \operatorname{sgn} y \gamma_{,y} = 0. \quad (29)$$

Thus, integrating eqn. (29) yields the trace γ as

$$\gamma = -\frac{1}{6k^2} \xi_{,\mu}^\mu (e^{\pm 2k|y|} - 1), \quad (30)$$

where we have used the residual gauge freedom to impose $\gamma = 0$ on the brane. We see from eqn. (30) that γ is unbounded for the RS background, if $\xi_{,\mu}^\mu \neq 0$. Moreover, this observation is independent of whether we use the residual gauge freedom as indicated or not. We shall see later that we do not have the choice of setting $\xi^\mu = 0$, if a matter perturbation is present on the brane. This indicates that, in the RS background, the linear approximation is not consistent. However, this might be an artifact of the particular choice of gauge. For further discussion of this problem, see [15, 28].

Finally, substituting eqns. (14), (28) and (30) into the equation of motion (27), we obtain the equation

$$\begin{aligned} \square \gamma_{\nu\rho} + \partial_y (e^{\mp 2k|y|} \gamma_{\nu\rho,y}) \mp 2k \operatorname{sgn} y e^{\mp 2k|y|} \gamma_{\nu\rho,y} - \xi_{\nu,\rho} - \xi_{\rho,\nu} \\ + \frac{1}{3} \eta_{\nu\rho} \xi_{,\mu}^\mu + \frac{1}{6k^2} (e^{\pm 2k|y|} - 1) \xi_{,\mu\nu\rho}^\mu \pm \frac{2}{3k} \delta(y) \eta_{\nu\rho} \xi_{,\mu}^\mu = -16\pi \delta(y) t_{\nu\rho}. \end{aligned} \quad (31)$$

We can take the trace of eqn. (31) and find

$$\xi_{,\mu}^\mu = \mp 8\pi k t, \quad (32)$$

where $t = t_\mu^\mu$. Thus, as indicated earlier, the four-divergence $\xi_{,\mu}^\mu$ is fixed by the content of matter perturbation on the brane.

As the next step, we consider the discontinuity of eqn. (31) at $y = 0$. One easily finds

$$-16\pi t_{\nu\rho} = \gamma_{\nu\rho,y}|_{y=+0} - \gamma_{\nu\rho,y}|_{y=-0} \pm \frac{2}{3k} \eta_{\nu\rho} \xi_{,\mu}^\mu. \quad (33)$$

However, as the perturbation is symmetric around the brane, we need only look for solutions which are even in y . Therefore, we can consider eqn. (31) in the region $y > 0$, and eqns. (33) and (32) provide the Neumann boundary condition

$$\gamma_{\nu\rho,y}|_{y=+0} = -8\pi \left(t_{\nu\rho} - \frac{1}{3} \eta_{\nu\rho} t \right). \quad (34)$$

Consider now eqn. (31) for $y > 0$. First, let us choose the vector ξ^μ as

$$\xi_\mu = \mp 8\pi k \frac{1}{\square} \partial_\mu t, \quad (35)$$

which is consistent with the condition (32). Then, we shall write

$$\gamma_{\nu\rho} = \pm \frac{4\pi}{3k} \left[\frac{1}{\square} t_{,\nu\rho} (e^{\pm 2ky} - 1) + 2k^2 \left(\eta_{\nu\rho} \frac{1}{\square} t - \frac{4}{\square^2} t_{,\nu\rho} \right) \right] + \tilde{\gamma}_{\nu\rho} \quad (36)$$

in order to obtain from eqn. (31) the following homogeneous equation for $\tilde{\gamma}_{\nu\rho}$:

$$\square \tilde{\gamma}_{\nu\rho} + \partial_y \left(e^{\mp 2ky} \tilde{\gamma}_{\nu\rho,y} \right) \mp 2k e^{\mp 2ky} \tilde{\gamma}_{\nu\rho,y} = 0. \quad (37)$$

Moreover, from eqn. (30) we find that the trace $\tilde{\gamma} \equiv 0$, and the Neumann boundary condition (34) yields

$$\tilde{\gamma}_{\nu\rho,y}|_{y=+0} = -8\pi \left(t_{\nu\rho} - \frac{1}{3} \eta_{\nu\rho} t + \frac{1}{3\square} t_{,\nu\rho} \right). \quad (38)$$

Notice that a trivial (zero) solution to the homogeneous equation (37) is not consistent with this boundary condition.

In order to solve eqn. (37), let us Fourier transform with respect to the brane coordinates and change variables to $z = e^{\pm 2ky}$. Then, eqn. (37) becomes

$$\left(z^2 \partial_z^2 - z \partial_z - \frac{p^2}{4k^2} z \right) \tilde{\gamma}_{\nu\rho} = 0, \quad (39)$$

whose solution can be expressed in terms of Bessel functions [34].

4 Newton's Law

In order to derive Newton's law on the brane, we have to look for a unique solution to the linearized Einstein equations in the presence of a static point source on the brane. In the last section, we presented the general formalism of linearized gravity. Let us now continue the solution for static point source. In order to obtain the Newtonian limit, we have to calculate $\gamma_{00}(x, 0)$, since the gravitational potential is given by

$$V = -\frac{m}{2} \gamma_{00}. \quad (40)$$

We need a second boundary condition for eqn. (37) in order to obtain a unique solution. We shall simply use

$$\tilde{\gamma}_{\nu\rho}|_{y=+\infty} = 0. \quad (41)$$

For static potentials, we have $p_0 = 0$, and therefore $p^2 \geq 0$ in eqn. (39). In fact, we need only consider $p^2 > 0$, as the solution for $p^2 = 0$ can be reconstructed as the limit $p^2 \rightarrow 0$. The solution of eqn. (39) for $p^2 > 0$ is

$$\tilde{\gamma}_{\nu\rho}(p, y) = c_{\nu\rho}(p) e^{\pm 2ky} \begin{cases} K_2(e^{\pm ky} |p|/k), \\ I_2(e^{\pm ky} |p|/k), \end{cases} \quad (42)$$

where the choice between the two possible solutions is dictated by eqn. (41). We easily see that this amounts to choosing the solution with the K function for the RS background (upper sign), and the solution with the I function for the alternative background.

Moreover, from eqn. (42) we find the first derivative,

$$\tilde{\gamma}_{\nu\rho,y}(p, y) = \pm |p| c_{\nu\rho}(p) e^{\pm 3ky} \begin{cases} -K_1(e^{\pm ky}|p|/k), \\ I_1(e^{\pm ky}|p|/k), \end{cases} \quad (43)$$

which, combined with the boundary condition (38), yields the coefficients

$$c_{\nu\rho} = \frac{8\pi}{|p|} \left[t_{\nu\rho} - \frac{1}{3} t \left(\eta_{\nu\rho} - \frac{p_\nu p_\rho}{p^2} \right) \right] \begin{cases} [K_1(|p|/k)]^{-1} & \text{for } \sigma < 0, \\ [I_1(|p|/k)]^{-1} & \text{for } \sigma > 0. \end{cases} \quad (44)$$

Thus, inserting eqns. (42) and (44) into eqn. (36), we obtain the solution for the metric perturbation

$$\begin{aligned} \gamma_{\nu\rho}(p, y) = & \pm \frac{4\pi}{3k} \left[\frac{p_\nu p_\rho}{p^2} (e^{\pm 2ky} - 1) - 2k^2 \left(\frac{\eta_{\nu\rho}}{p^2} - \frac{4p_\nu p_\rho}{p^4} \right) \right] t \\ & + \frac{8\pi}{|p|} \left[t_{\nu\rho} - \frac{1}{3} t \left(\eta_{\nu\rho} - \frac{p_\nu p_\rho}{p^2} \right) \right] \begin{cases} e^{2ky} \frac{K_2(e^{ky}|p|/k)}{K_1(|p|/k)} & \text{for } \sigma < 0, \\ e^{-2ky} \frac{I_2(e^{-ky}|p|/k)}{I_1(|p|/k)} & \text{for } \sigma > 0. \end{cases} \end{aligned} \quad (45)$$

Let us now use a static point source, $t_{00}(p) = 2\pi\delta(p_0)a$, where $a = M/M_{Pl(5)}^3$, and solve for $\gamma_{00}(x, 0)$. Consider first the RS background. We can use the recursion formula for modified Bessel functions,

$$K_2(z) = \frac{2}{z} K_1(z) + K_0(z),$$

in order to separate the divergent term for $|p| \rightarrow 0$ in the Fourier integral. Then, from eqn. (45) we find

$$\begin{aligned} \gamma_{00}(x, 0) = & -\frac{2ka}{3r} + \frac{8ka}{3r} + \lim_{y \rightarrow 0} \int \frac{d^3p}{(2\pi)^3} e^{-ip \cdot x} \frac{16\pi a}{3|p|} \frac{K_0(e^{ky}|p|/k)}{K_1(|p|/k)} \\ = & \frac{2ka}{r} + \frac{8a}{3\pi r^2} \lim_{y \rightarrow 0} \int_0^\infty ds \sin s \frac{K_0(e^{ky}s/kr)}{K_1(s/kr)}. \end{aligned} \quad (46)$$

It is interesting to note that the inhomogeneous terms of the solution would yield Newton's law with a wrong sign, but the homogeneous part, whose presence is necessary because of the Neuman boundary condition, takes care of this. In fact, the importance of the boundary conditions for obtaining the gravity on the brane has already been pointed out in [35]. Moreover, we observe that the integral in eqn. (46) is well-defined only for $y > 0$. It is for this reason that we take the limit $y \rightarrow 0$ after the integration. The non-zero y acts as a regulator of the integral for large s . The second term in eqn. (46) is a correction to Newton's law, which we shall demonstrate in a moment.

Let us consider now the alternative background. From eqn. (45) we obtain

$$\begin{aligned} \gamma_{00}(x, 0) = & \frac{2ka}{3r} + \lim_{y \rightarrow 0} \int \frac{d^3p}{(2\pi)^3} e^{-ip \cdot x} \frac{16\pi a}{3|p|} \frac{I_2(e^{-ky}|p|/k)}{I_1(|p|/k)} \\ = & \frac{2ka}{3r} + \frac{8a}{3\pi r^2} \lim_{y \rightarrow 0} \int_0^\infty ds \sin s \frac{I_2(e^{-ky}s/kr)}{I_1(s/kr)}. \end{aligned} \quad (47)$$

The first term in eqn. (47) represents Newton's law, and the second term corrections, as we shall demonstrate now.

Consider an integral of the form $\int_0^\infty ds \sin s f(s/z, y)$, where f is a differentiable and integrable function. From [34] we know that the integrands in eqns. (46) and (47) satisfy this property for any $y > 0$. Given the integrability of f , we can rewrite the integral as

$$\int_0^\infty ds \sin s f(s/z, y) = \sum_{k=0}^\infty \int_{-\pi}^\pi ds (-\sin s) f\left(\frac{\pi(2k+1) + s}{z}, y\right). \quad (48)$$

At this point, we can take the limit $y \rightarrow 0$, and we shall write $f(x, 0) = f(x)$. For large z , the argument of f will change little in one period of the sin function, and we can write

$$\begin{aligned} \int_0^\infty ds \sin s f(s/z) &\approx - \sum_{k=0}^\infty \int_{-\pi}^\pi ds \sin s \left[f\left(\frac{\pi(2k+1)}{z}\right) + \frac{s}{z} f'\left(\frac{\pi(2k+1)}{z}\right) \right] \\ &\approx - \sum_{k=0}^\infty \frac{2\pi}{z} f'\left(\frac{\pi(2k+1)}{z}\right). \end{aligned}$$

Here, we observe that the $z \rightarrow \infty$ limit exists, namely it is just the integral

$$- \int_0^\infty dx f'(x) = f(0) - f(\infty).$$

For both integrands under consideration we have $f(0) = 0$ and $f(\infty) = 1$ (for $y > 0$ we would have the stronger $f(\infty) = 0$). Thus, we find that the second terms in eqns. (46) and (47) go to zero at least as fast as $1/r^2$ for large r and are in fact corrections to Newton's law.

Finally, from eqn. (40) we obtain the gravitational potential

$$V(r) = -\frac{kmM}{M_{Pl(5)}^3} \begin{cases} 1/r + \mathcal{O}(1/r^2) & \text{for } \sigma < 0, \\ 1/(3r) + \mathcal{O}(1/r^2) & \text{for } \sigma > 0. \end{cases} \quad (49)$$

Thus, one can read off the four-dimensional Planck masses as

$$M_{Pl(4)}^2 = \begin{cases} M_{Pl(5)}^3/k & \text{for } \sigma < 0, \\ 3M_{Pl(5)}^3/k & \text{for } \sigma > 0. \end{cases} \quad (50)$$

Our result for the RS background is in agreement with the result given in [36].

5 Graviton Modes

It is our objective in this section to study graviton modes. The equation of motion for gravitons in the gauge $\delta g_{a5} = 0$ is the homogeneous equation (37) with the boundary condition

$$\gamma_{\nu\rho, y}|_{y=0} = 0. \quad (51)$$

This boundary condition stems from eqn. (34) and implies that we consider only modes which are even in y . This would be natural for the orbifold S^1/\mathbb{Z}_2 , but in general one would have to match a solution for $y > 0$ with a solution for $y < 0$.

Let us now give the graviton solutions. As we consider only modes which are even in y , we restrict ourselves to $y \geq 0$. Using eqn. (39), the solutions to eqn. (37) satisfying the boundary condition (51) are

$$\gamma_{\mu\nu}(p, y) = c_{\mu\nu}(p) \begin{cases} e^{\pm 2ky} [\text{I}_2(\alpha e^{\pm ky})\text{K}_1(\alpha) + \text{K}_2(\alpha e^{\pm ky})\text{I}_1(\alpha)] & \text{for } p^2 > 0, \\ 1 & \text{for } p^2 = 0, \\ e^{\pm 2ky} [\text{J}_2(\alpha e^{\pm ky})\text{N}_1(\alpha) - \text{N}_2(\alpha e^{\pm ky})\text{J}_1(\alpha)] & \text{for } p^2 < 0. \end{cases} \quad (52)$$

Here, we have set $\alpha = \sqrt{|p^2|}/k$.

Let us discuss the normalizability of these modes. For the RS background, one finds that space-like modes ($p^2 > 0$) are not normalizable, since I_2 diverges as e^{ey} for large y . On the other hand, zero-like and time-like modes are normalizable.

On the other hand, for the alternative background the y integral diverges in all three cases just as the volume integral. Thus, just as in flat space, we might assume that we can form wave packets which describe normalizable wave functions. Alternatively, one might regularize integrals like the norm of wavefunctions or the effective action by dividing by the total volume of space. We would like to leave these points open to further research.

6 Conclusions

In this paper, we have studied the geodesics and derived Newton's law for the Randall-Sundrum and an alternative brane background. We found that matter will be expelled from the brane in the RS background. A similar behaviour was observed by Rubakov *et al.* [37] in a different context. Therefore, the RS background is classically unstable, and it must be supplemented with a mechanism for confinement of matter. On the other hand, gravity provides a natural mechanism for the trapping of matter in the alternative background. It would be interesting to study a realization of the alternative background in supergravity. The RS background in supergravity has been discussed in [8, 23, 24, 25, 26].

Our derivation of gravity on the brane revealed the validity of Newton's law to leading order in both backgrounds, but the corresponding Planck masses on the brane are different. Moreover, we found exact formulas, eqns. (46) and (47), for the corrections.

In the RS background there are no normalizable space-like modes, which certainly is in favour of this background. For the alternative background, all modes are non-integrable, but a thorough discussion of the confinement or non-confinement of gravity remains an open problem. It seems that the confinement of gravity cannot be discussed separately from the confinement of matter.

Acknowledgements

This research was partly supported by NSERC. I. V. is grateful to the Physics Department of Simon Fraser University for its kind hospitality. Moreover, we are grateful to I. Ya. Aref'eva

and M. G. Ivanov for their help in the early stage of this work and to A. Linde and V. A. Rubakov for their critical remarks.

Appendix

We state here some expressions for the connections and curvature up to first order in the perturbations $\gamma_{\mu\nu}$. The only non-zero connections for the metric g_{ab} are

$$\Gamma^\mu{}_{\nu\lambda} = \frac{1}{2} (\gamma^\mu{}_{\nu,\lambda} + \gamma^\mu{}_{\lambda,\nu} - \gamma_{\nu\lambda}{}^{,\mu}), \quad (53)$$

$$\Gamma^y{}_{\nu\lambda} = \pm k \operatorname{sgn} y \, g_{\nu\lambda} - \frac{1}{2} e^{\mp 2k|y|} \gamma_{\nu\lambda,y}, \quad (54)$$

$$\Gamma^\nu{}_{\lambda y} = \Gamma^\nu{}_{y\lambda} = \mp k \operatorname{sgn} y \, \delta_\lambda^\nu + \frac{1}{2} \gamma^\nu{}_{\lambda,y}. \quad (55)$$

Moreover, we find from eqn. (18)

$$H_\nu^\mu = \mp k \operatorname{sgn} y \, \delta_\nu^\mu + \frac{1}{2} \gamma^\mu{}_{\nu,y}, \quad (56)$$

and some expressions for the curvatures are ($\square = \eta^{mu\nu} \partial_\mu \partial_\nu$)

$$\hat{R}_{\nu\rho} = \frac{1}{2} (\gamma^\mu{}_{\nu,\rho\mu} + \gamma^\mu{}_{\rho,\nu\mu} - \gamma_{\nu\rho}{}^{,\mu} - \square \gamma_{\nu\rho}), \quad (57)$$

$$\hat{R} = e^{\pm 2k|y|} (\gamma^{\mu\nu}{}_{,\mu\nu} - \square \gamma), \quad (58)$$

$$R^y{}_{\nu y\rho} = -k^2 g_{\nu\rho} \pm 2k\delta(y)g_{\nu\rho} + e^{\mp 2k|y|} \left(\pm k \operatorname{sgn} y \, \gamma_{\nu\rho,y} - \frac{1}{2} \gamma_{\nu\rho,yy} \right), \quad (59)$$

$$R_{\mu\rho} = \hat{R}_{\mu\rho} + H_\mu^\nu H_{\nu\rho} - H H_{\mu\rho} + R^y{}_{\mu y\rho}, \quad (60)$$

$$R = \hat{R} - 20k^2 \pm 16k\delta(y) \pm 5k \operatorname{sgn} y \, \gamma_{,y} - \gamma_{,yy}. \quad (61)$$

References

- [1] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83**, 4690 (1999).
- [2] V. A. Rubakov and M. E. Shaposhnikov, *Phys. Lett.* **125B**, 136 (1983).
- [3] M. Visser, *Phys. Lett.* **B159**, 22 (1985).
- [4] K. Akama, *An Early Proposal of "Brane World"*, hep-th/0001113.
- [5] N. Arkani-Hamed, S. Dimopoulos, G. Dvali and N. Kaloper, *Phys. Rev. Lett.* **84**, 586 (2000).
- [6] W. D. Goldberger and M. B. Wise, *Phys. Rev. Lett.* **83**, 4922 (1999).

- [7] M. Gogberashvili, *Gravitational Trapping for Extended Extra Dimension*, [hep-ph/9908347](#).
- [8] K. Behrndt and M. Cvetič, *Supersymmetric Domain-Wall World from $D = 5$ Simple Gauged Supergravity*, [hep-th/9909058](#).
- [9] E. Halyo, *Localized Gravity on Branes in anti-de Sitter Spaces*, [hep-th/9909127](#).
- [10] A. Chamblin and G. W. Gibbons, *Supergravity on the Brane*, [hep-th/9909130](#).
- [11] O. deWolfe, D. Z. Freedman, S. S. Gubser and A. Karch, *Modeling the Fifth Dimension with Scalars and Gravity*, [hep-th/9909134](#).
- [12] I. Y. Aref'eva, *High Energy Scattering in the Brane-World and Black Hole Production*, [hep-th/9910269](#).
- [13] C. Csaki, M. Graesser, L. Randall and J. Terning, *Cosmology of Brane Models with Radion Stabilization*, [hep-ph/9911406](#).
- [14] C. Charmousis, R. Gregory and V. A. Rubakov, *Wave Function of the Radion in a Brane World*, [hep-th/9912160](#).
- [15] J. Garriga and T. Tanaka, *Gravity in the Brane-World*, [hep-th/9911055](#).
- [16] A. G. Cohen and D. B. Kaplan, *Phys. Lett.* **B470**, 52 (1999).
- [17] E. Verlinde and H. Verlinde, *RG-flow, Gravity and the Cosmological Constant*, [hep-th/9912018](#).
- [18] J. Lykken and L. Randall, *The Shape of Gravity*, [hep-th/9908076](#).
- [19] N. Kaloper, *Phys. Rev. D* **60**, 123506 (1999).
- [20] M. Sasaki, T. Shiromizu and K. Maeda, *Gravity, Stability and Energy Conservation on the Randall-Sundrum Brane-World*, [hep-th/9912223](#).
- [21] M. G. Ivanov and I. V. Volovich, *Metric Fluctuations in Brane Worlds*, [hep-th/9912242](#).
- [22] Y. S. Myung and G. Kang, *Comment on "Metric Fluctuations in Brane Worlds"*, [hep-th/0001003](#).
- [23] D. Youm, *Solitons in Brane Worlds*, [hep-th/9911218](#).
- [24] D. Youm, *A Note on Solitons in Brane Worlds*, [hep-th/0001166](#).
- [25] D. Youm, *Solitons in Brane Worlds II*, [hep-th/0001018](#).
- [26] R. Kallosh and A. Linde, *Supersymmetry and the Brane World*, [hep-th/0001071](#).
- [27] C. Csaki, J. Erlich, T. J. Hollowood and Y. Shirman, *Universal Aspects of Gravity Localized on Thick Branes*, [hep-th/0001033](#).

- [28] S. B. Giddings, E. Katz and L. Randall, *Linearized Gravity in Brane Backgrounds*, hep-th/0002091.
- [29] R. Gregory, V. A. Rubakov and S. M. Sibiryakov, *Opening up Extra Dimensions at Very Large Distances*, hep-th/0002072.
- [30] M. Gogberashvili, *Hierarchy Problem in the Shell-Universe Model*, hep-ph/9812296.
- [31] M. Gogberashvili, *Europhys. Lett.* **49**, 396 (1998).
- [32] B. Bajc and G. Gabadadze, *Localization of Matter and Cosmological Constant on a Brane in Anti de Sitter Space*, hep-th/9912232.
- [33] C. W. Misner, K. S. Thorne and J. A. Wheeler, *Gravitation*, Freeman, San Francisco (1973).
- [34] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, Academic Press, New York, 5th edn. (1994).
- [35] R. Dick and D. Mikulovicz, *Gravity and the Newtonian Limit in the Randall-Sundrum Model*, hep-th/0001013.
- [36] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83**, 3370 (1999).
- [37] V. A. Rubakov, *private communication*.